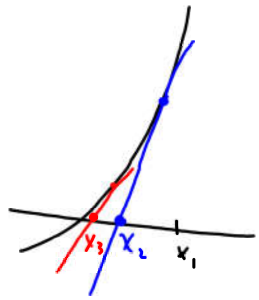
More 4.5 - Newton's Method



Method for finding zeros of a function.

1. Start with a guess: x_1
2. Do a linearization with center x_1
3. Find the zero of your linearization → next guess!
4. Repeat!

$$L(x) = f(a) + f'(a)(x-a)$$

$$L(x) = f(x_1) + f'(x_1)(x - x_1)$$

$$0 = f(x_1) + f'(x_1)(x - x_1)$$

$$-f(x_1) = f'(x_1)(x - x_1)$$

$$\frac{-f(x_1)}{f'(x_1)} = x - x_1$$

$$x_1 - \frac{f(x_1)}{f'(x_1)} = x_2$$

$$x_n - \frac{f(x_n)}{f'(x_n)} = x_{n+1}$$

Ex) Solve $x^3 + 3x + 1 = 0$ accurate to the 8th decimal place. Let $x_1 = -0.3$

On your calculator:

$Y1 = f(x) = x^3 + 3x + 1$

$Y2 = f'(x) = 3x^2 + 3$

$-0.3 \xrightarrow{\text{STO}}$ Alpha X

-0.322324159

-0.3221853603

-0.3221853546

-0.3221853546

$\boxed{x = -0.32218535}$

Limitations of Newton's Method: p237

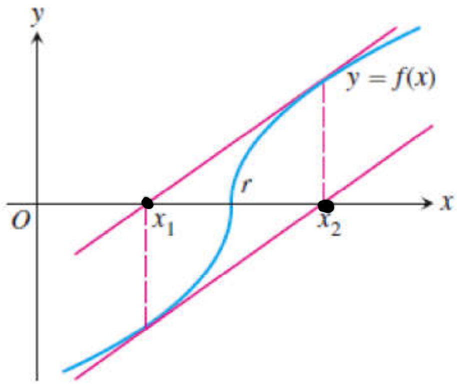


Figure 4.51 The graph of the function

$$f(x) = \begin{cases} -\sqrt{r-x}, & x < r \\ \sqrt{x-r}, & x \geq r. \end{cases}$$

If $x_1 = r - h$, then $x_2 = r + h$. Successive approximations go back and forth between these two values, and Newton's method fails to converge.

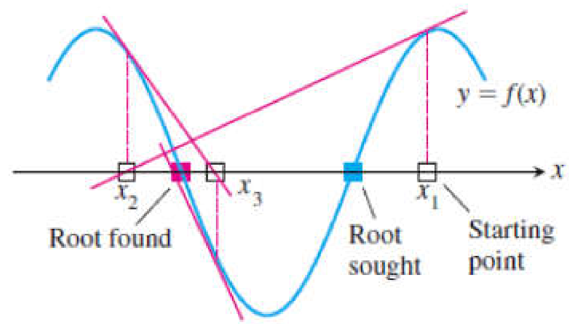


Figure 4.52 Newton's method may miss the zero you want if you start too far away.

HW: p242 #7, 8, $\boxed{15, 16, 18}$, 71

→ graph on your calculator to get x_1

16. −1.452627 and 1.164035

45) $f(0)=1 \quad f'(x)=\cos(x^2)$

a) $L(x)=f(a)+f'(a)(x-a)$

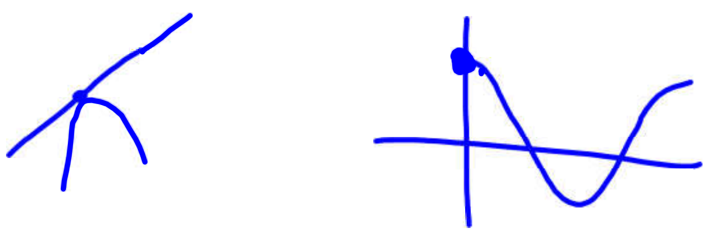
$=f(0)+f'(0)(x-0)$

$=1+1(x-0)$

$=1+x$

b) $f(0.1)\approx L(0.1)=1+0.1=1.1$

c)



p242

(2) $L(x) = -\frac{4}{5}x + \frac{9}{5}$

$L(x) = 4.92$ $f(x) \approx 4.9204$